	$-\alpha$	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3}{2}\pi-\alpha$	$\frac{3}{2}\pi+\alpha$	
sin	$-\sin\alpha$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$\sin^2\alpha = \frac{\tan^2\alpha}{1+\tan^2\alpha}$
cos	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$\cos^2\alpha = \frac{1}{1+\tan^2\alpha}$
tan	$-\tan\alpha$	$\cot\alpha$	$-\cot\alpha$	$-\tan\alpha$	$\tan\alpha$	$\cot\alpha$	$-\cot\alpha$	
cot	$-\cot\alpha$	$\tan\alpha$	$-\tan\alpha$	$-\cot\alpha$	$\cot\alpha$	$\tan\alpha$	$-\tan\alpha$	

$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$	$\tan(\alpha\pm\beta) = \frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$	$\cot(\alpha\pm\beta) = \frac{\cot\alpha\cot\beta\mp 1}{\cot\alpha\pm\cot\beta}$
$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$	$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$	$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$	$\cot\frac{\alpha}{2} = \frac{\sin\alpha}{1-\cos\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$	$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$	$\cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$

$\sin\alpha = \frac{2t}{1+t^2}$	$\arcsin : [-1,1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	crescente	$\lim_{x\to 0}\frac{\sin x}{x} = 1$	$\lim_{x\to 0}\frac{\log_a(1+x)}{x} = \log_a e$
$\cos\alpha = \frac{1-t^2}{1+t^2}$	$\arccos : [-1,1] \to [0,\pi]$	decrescente	$\lim_{x\to 0}\frac{\tan x}{x} = 1$	$\lim_{x\to 0}\frac{e^x-1}{x} = 1$
$\tan\alpha = \frac{2t}{1-t^2}$	$\arctan : \mathbb{R} \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	crescente	$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$	$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$
$\cot\alpha = \frac{1-t^2}{2t}$	$\text{arccot} :]0,\pi[\to \mathbb{R}$	crescente	$\lim_{x\to 0}\frac{\ln(1+x)}{x} = 1$	$\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$

$\lim_n \sqrt[n]{n} = 1 \quad \lim_n \frac{n^a}{b^n} = 0 \quad \lim_n \frac{a^n}{n!} = 0 \quad \lim_n \frac{a^{\frac{1}{n}}-1}{\frac{1}{n}} = \ln a \quad \lim_n \frac{\ln(1+\frac{1}{n})}{\frac{1}{n}} = 1 \quad \lim_n \frac{\log_a(1+\frac{1}{n})}{\frac{1}{n}} = \log_a e$

$f(x)$	$f'(x)$	$\int f(x)dx$	$f(x)$	$f'(x)$	$\int f(x)dx$	$f(x)$	$f'(x)$	$\int f(x)dx$
a^x	$a^x\ln a$	$\frac{a^x}{\ln a}$	$\tan x$	$\frac{1}{\cos^2 x}$	$-\ln\lvert\cos x\rvert$	$\cot x$	$\frac{-1}{\sin^2 x}$	$\ln\lvert\sin x\rvert$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	$x\arcsin x + \sqrt{1-x^2}$	$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$	$x\arccos x - \sqrt{1-x^2}$	$\arctan x$	$\frac{1}{1+x^2}$	$x\arctan x - \ln\sqrt{1+x^2}$
$\text{setsinh}x$	$\frac{1}{\sqrt{1+x^2}}$	?	$\text{setcosh}x$	$\frac{1}{\sqrt{x^2-1}}$	?	$\text{settanh}x$	$\frac{1}{1-x^2}$	$\frac{1}{2}\left[(1-x^2)\text{settanh}x - x)\right]$

!!! Attezione: $\int\frac{1}{1-x^2}dx = \begin{cases}\text{settanh}x + c & \text{se } \lvert x\rvert < 1\\ \text{arccoth}x + c & \text{se } \lvert x\rvert > 1\end{cases}$

$\int[f(x)]^k f'(x)dx = \frac{[f(x)]^{k+1}}{k+1} + c$	$\int\frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert + c$	$\int e^{f(x)}f'(x)dx = e^{f(x)} + c$
$\int a^{f(x)}f'(x)dx = a^{f(x)}\log_a e + c$	$\int\cos f(x)f'(x)dx = \sin f(x) + c$	$\int\sin f(x)f'(x)dx = -\cos f(x) + c$
$\int\frac{f'(x)}{\cos^2 f(x)}dx = \tan f(x) + c$	$\int\frac{f'(x)}{\sin^2 f(x)}dx = -\cot f(x) + c$	$\int\frac{f'(x)}{\sqrt{1-[f(x)]^2}}dx = \arcsin f(x) + c$
$\int\frac{f'(x)}{1+[f(x)]^2}dx = \arctan f(x) + c$	$\int\frac{f'(x)}{\sqrt{1+[f(x)]^2}}dx = \text{setsinh}f(x) + c$	$\int\frac{f'(x)}{\sqrt{[f(x)]^2-1}}dx = \text{setcosh}f(x) + c$
$\int\frac{f'(x)}{1-[f(x)]^2}dx = \text{settanh}f(x) + c$	$\int\frac{1}{\sin x}dx = \log\left\lvert\tan\frac{x}{2}\right\rvert + c$	$\int\frac{1}{\cos x}dx = \log\left\lvert\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right\rvert + c$

$\int\frac{1}{1+\sin x}dx = \int\frac{1-\sin x}{1-\sin^2 x}dx = \int\frac{1-\sin x}{\cos^2 x}dx = \tan x - \frac{1}{\cos x} + c$; $\int\frac{1}{\sin x\cos x}dx = \int\frac{1}{\tan x\cos^2 x}dx = \log\lvert\tan x\rvert + c$;

$\int\frac{1}{\sin^2 x\cos^2 x}dx = \int\frac{1}{\left[\frac{\sin 2x}{2}\right]}dx = -2\cot 2x + c$; $\int\frac{1}{x(1+x)}dx = \int\frac{1+x-x}{x(1+x)}dx = \log\lvert x\rvert - \log\lvert 1+x\rvert + c$;

$\int\sin^2 x dx = \int\frac{1-\cos 2x}{2}dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$; $\int\cos^2 x dx = \int\frac{1+\cos 2x}{2}dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$;

$\int\sin ax\cos ax dx = \int\frac{1}{2}\sin 2ax dx = -\frac{\cos 2ax}{4a} + c$; $\int\sin^3 x dx = \int(1-\cos^2 x)\sin x dx = -\cos x + \frac{\cos^3 x}{3} + c$;

$\int\cos^3 x dx = \int(1-\sin^2 x)\cos x dx = \sin x - \frac{\sin^3 x}{3} + c$; $\int\cot^2 x dx = \int\frac{1-\sin^2 x}{\sin^2 x}dx = -\cot x - x + c$;

$\int\frac{x}{\sin^2 x}dx = parti = -x\cot x + \log\lvert\sin x\rvert + c$; $\int\frac{x}{\cos^2 x}dx = parti = x\tan x + \log\lvert\cos x\rvert + c$;

$\int x\tan^2 x dx = \int x\frac{1-\cos^2 x}{\cos^2 x} = x\tan x + \log\lvert\cos x\rvert - \frac{x^2}{2} + c$; $I_n = \int\frac{1}{(1+x^2)^n}dx = \frac{1}{2n-1}\left[(2n-3)I_{n-1} + \frac{x}{(1+x^2)^{n-1}}\right]$;

$\int x\arctan^2 x dx = parti = \frac{1}{2}x^2\arctan^2 x - x\arctan x + \log\sqrt{1+x^2} + \frac{1}{2}\arctan^2 x + c$;

Integrale	Sostituzione	Soluzione	Integrale	Sostituzione	Soluzione
$\int\sqrt{a^2-x^2}dx$	$x = a\sin t$	$\frac{a^2}{2}\arcsin\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2}$	$\int\sqrt{2^x-1}dx$	$t = 2^x - 1$	$\frac{2}{\ln 2}\left(\sqrt{2^x-1} - \arctan\sqrt{2^x-1}\right)$
$\int\sin\log x dx$	$t = \log x$	$\frac{x}{2}\left[\sin(\log x) - \cos(\log x)\right]$	$\int\frac{1}{x\sqrt{x+4}}dx$	$t = \sqrt{x+4}$	$\frac{1}{2}\left(\log\frac{\lvert\sqrt{x+4}-2\rvert}{\log\sqrt{x+4}+2}\right)$
$\int x\sqrt{\frac{1-x}{1+x}}dx$	$t = \sqrt{\frac{1-x}{1+x}}$	$\arctan t - \frac{3t}{t^2+1} + \frac{2t}{(t^2+1)^2}$	$\int\frac{1}{x+\sqrt{1+x^2}}dx$	$t = x + \sqrt{1+x^2}$	$\frac{1}{2}\log\lvert t\rvert - \frac{1}{8t^2}$

$\sum_{n=1}^{\infty}x^{n-1}$	oscilla per $x \leq 1$ converge a $(1-x)^{-1}$ per $-1 < x < 1$ diverge a $+\infty$ per $x \geq 1$	$\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$	converge a 1	$\sum_{n=1}^{\infty}\frac{1}{n^x}$	diverge per $x \leq 1$ converge per $x > 1$

$\sum_{n=1}^{\infty}a_n$ converge $\Rightarrow \lim_n a_n = 0$; $a_n \geq 0\ \forall n \Rightarrow \sum_{n=1}^{\infty}a_n$ regolare;

$\lim_n\frac{a_{n+1}}{a_n} =$	$l > 1 \Rightarrow$ diverge $l = 1 \Rightarrow$? $l < 1 \Rightarrow$ converge	$\lim_n\sqrt[n]{a_n} =$	$l > 1 \Rightarrow$ diverge $l = 1 \Rightarrow$? $l < 1 \Rightarrow$ converge	$\lim_n n\left(\frac{a_n}{a_{n+1}} - 1\right) =$	$l > 1 \Rightarrow$ converge $l = 1 \Rightarrow$? $l < 1 \Rightarrow$ diverge

$\exists x > 1 : \lim_n n^x a_n < +\infty \Rightarrow$ converge; $\exists x \leq 1 : \lim_n n^x a_n > 0 \Rightarrow$ diverge a $+\infty$;

$\sum_{n=1}^{+\infty}(-1)^n a_n,\ a_n \geq 0\ \forall n$: $\{a_n\}\ nc,\ lim_n a_n = 0 \Rightarrow$ converge; $\{a_n\}\ nc,\ lim_n a_n \neq 0 \Rightarrow$ oscilla; $\{a_n\}\ nd \Rightarrow$ oscilla;

$\sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!} =$	$e^x \ \forall x \in \mathbb{R}$		Teo. derivazione per serie
		Hp.	(1) $\sum_{n=1}^{\infty} f_n(x),\ f_n : (a,b) \to \mathbb{R} \ \in \ C^1((a,b))$, $x_0 \in (a,b)$, (1)$c.p.inx_0$, (2) $\sum_{n=1}^{\infty} f'_n(x) \xrightarrow{u} g(x) : (a,b) \to \mathbb{R}$;
$\sum_{n=1}^{\infty} \frac{x^n}{n}$	converge per $-1 \leq x < 1$ diverge a $+\infty$ per $x > 1$ oscilla per $x < -1$	Ts.	(1) $\xrightarrow{u} S(x) : (a,b) \to \mathbb{R} \ \in \ C^1((a,b))$, $S'(x) = g(x) \ \forall x \in (a,b)$;

Teo. di integazione per serie

Hp. (1) $\sum_{n=1}^{\infty} f_n(x),\ f_n : [a,b] \to \mathbb{R} \ \in \ C^1([a,b])$, (1) $\xrightarrow{u} S : [a,b] \to \mathbb{R}$

Ts. $\sum_{n=1}^{\infty} \int_a^b f_n(x)dx = \int_a^b S(x)dx$

Serie di potenze: $\sum_{n=1}^{\infty} a_{n-1}(x-x_0)^{n-1}$, $r = \sup X = \sup\{h \geq 0 \mid \sum_{n=1}^{\infty} |a_{n-1}|h^{n-1} \ conv.\}$, $I_c = (x_0 - r, x_0 + r)$.
$r = \lim_n \left|\frac{a_{n-1}}{a_n}\right| = \left(\lim_n \sqrt[n]{a_n}\right)^{-1}$.

Sviluppo in serie di Taylor: $f(c) + \frac{f'(c)}{1!}(x-c)^1 + \cdots + \frac{f^{(n-1)}(c)}{(n-1)!}(x-c)^{n-1} + \cdots$

Sviluppo in serie di McLaurin: $f(0) + \frac{f'(0)}{1!}x + \cdots + \frac{f^{(n-1)}(0)}{(n-1)!}x^{n-1} + \cdots$

$f^{(n)}(x) \leq kM^n \ \forall x \in (c,x) \ o \ (x,c) \ \forall n \ \Rightarrow \ f$ sviluppabile nel punto x in S.T. di centro c.

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$	$\sin x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$	$\cos x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$	$\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$	$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$
$(1+x)^\alpha = \sum_{n=0}^{\infty} \binom{a}{n} x^n$	$\frac{1}{1+x} = \sum_{n=0}^{\infty}(-1)^n x^n$	$\frac{1}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n x^{2n}$	$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+1}$	$\arctan x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$

$\lim_{(x,y)\to(x_0,y_0)} = l \Leftrightarrow \forall \varepsilon > 0 \ \exists \delta(\varepsilon) > 0 \ : \ \forall (x,y) \ : \ 0 < d((x,y),(x_0,y_0)) < \delta(\varepsilon)$ si abbia $|f(x,y) - l| < \varepsilon$;
$(x_0,y_0) \in DX,\ f \in C^0(\{(x_0,y_0\}) \Leftrightarrow \forall \varepsilon > 0 \ \exists \delta(\varepsilon) > 0 \ : \ \forall (x,y) \ 0 < d((x,y),(x_0,y_0)) < \delta(\varepsilon)$ si abbia $|f(x,y) - f(x_0,y_0)| < \varepsilon$;
$f \ u.c. \ in \ X \Leftrightarrow \forall \varepsilon > 0 \ \exists \delta(\varepsilon) > 0 \ : \ \forall P, P' \ : \ d(P,P') < \delta(\varepsilon)$ si abbia $|f(P) - f(P')| < \varepsilon$; $\frac{\partial f}{\partial x}(x_0,y_0) = \lim_{x\to x_0} \frac{f(x,y_0) - f(x_0,y_0)}{x - x_0}$;
$\frac{\partial f}{\partial y}(x_0,y_0) = \lim_{y\to y_0} \frac{f(x_0,y) - f(x_0,y_0)}{y - y_0}$; $df = \frac{\partial f}{\partial x}(x_0,y_0) \cdot h + \frac{\partial f}{\partial y}(x_0,y_0) \cdot k$; $\Delta f = f(x_0+h, y_0+k) - f(x_0,y_0)$;
f differenziabile in $(x_0,y_0) \Leftrightarrow \lim_{(h,k)\to(0,0)} \frac{\Delta f - df}{\sqrt{h^2+k^2}} = 0$; $\nabla f = \left(\frac{\partial f}{\partial x}(x,y), \frac{\partial f}{\partial y}(x,y)\right)$;

Teo. di Schwartz: Hp. $f : A \to \mathbb{R},\ A \ aperto$; $\exists \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial x \partial y}$ in tutti i punti di A; $P_0 \in A$; $\frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial x \partial y} \in C^0(\{P_0\})$;
Ts. $\frac{\partial^2 f}{\partial y \partial x} P_0 = \frac{\partial^2 f}{\partial x \partial y} P_0$;

Teo. del differenziale: Hp. $f : A \to \mathbb{R}$; $P_0 \in \mathring{A}$; $\exists \frac{\partial f}{\partial x} P_0$; $\exists \frac{\partial f}{\partial y}(x,y) \ \forall (x,y) \in I(P_0, \delta) \subseteq A$; $\frac{\partial f}{\partial y} \in C^0(\{P_0\})$;
Ts. f differenziabile in P_0;

Teo. del gradiente: Hp. $f : a \to \mathbb{R},\ A \ aperto,\ A \ internamente \ connesso$; $\exists \nabla f = 0 \ \forall (x,y) \in A$; Ts. f costante;